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6 Circular Motion

6-1 Centripetal Acceleration and Force

Period, Frequency, and Speed

Vocabulary **Period:** The time it takes for one full rotation or revolution of an object.

Vocabulary **Frequency:** The number of rotations or revolutions per unit time.

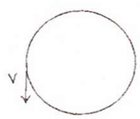
Period and frequency are reciprocals of each other. In other words,

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Since period is a measure of time, its SI unit is the **second**, while the unit for frequency is the reciprocal of this, or 1/second. Another way of writing 1/second is with the unit **hertz (Hz)**.

When an object spins in a circle, the distance it travels in one revolution is the circumference of the circle, $2\pi r$. The time it takes for one revolution is the period, T. Therefore,

$$\text{speed} = \frac{2\pi(\text{radius})}{\text{period}} \quad \text{or} \quad v = \frac{2\pi r}{T}$$



where v is called the **linear** or **tangential speed** because at any given time, the velocity is tangent to the circle as shown in the diagram. Although the velocity is constant in magnitude (speed), it is always changing direction.

Centripetal Acceleration and Centripetal Force

An object can move around in a circle with a constant speed yet still be accelerating because its direction is constantly changing. This acceleration, which is always directed in toward the center of the circle, is called **centripetal acceleration**. The magnitude of this acceleration is written as

$$\text{centripetal acceleration} = \frac{(\text{linear speed})^2}{\text{radius}} \quad \text{or} \quad a_c = \frac{v^2}{r}$$

If a mass is being accelerated toward the center of a circle, it must be acted upon by an unbalanced force that gives it this acceleration. This force, called the **centripetal force**, is always directed inward toward the center of the circle. The magnitude of this force is written as

$$\text{centripetal force} = (\text{mass})(\text{centripetal acceleration})$$

$$\text{or} \quad F_c = ma_c = \frac{mv^2}{r}$$

The units for centripetal acceleration and centripetal force are m/s^2 and N, respectively.

Solved Examples

Example 1: After closing a deal with a client, Kent leans back in his swivel chair and spins around with a frequency of 0.5 Hz. What is Kent's period of spin?

Given: $f = 0.5$ Hz

Unknown: $T = ?$
Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{0.5\text{ Hz}} = 2\text{ s}$

Example 2: Curtis' favorite disco record has a scratch 12 cm from the center that makes the record skip 45 times each minute. What is the linear speed of the scratch as it turns?

Solution: The record makes 45 revolutions every 60. seconds, so find the period of the record first.

$$T = \frac{60.\text{ s}}{45\text{ rev}} = 1.3\text{ s}$$

Given: $r = 12$ cm
$T = 1.3$ s

Unknown: $v = ?$
Original equation: $v = \frac{2\pi r}{T}$

Solve: $v = \frac{2\pi r}{T} = \frac{2\pi(12\text{ cm})}{1.3\text{ s}} = 58\text{ cm/s}$

Example 3: Missy's favorite ride at the Topsfield Fair is the rotor, which has a radius of 4.0 m. The ride takes 2.0 s to make one full revolution.
a) What is Missy's linear speed on the rotor?
b) What is Missy's centripetal acceleration on the rotor?

Solution: The ride takes 2.0 s to make one full revolution, so the period is 2.0 s.

a. *Given:* $r = 4.0$ m
$T = 2.0$ s

Unknown: $v = ?$
Original equation: $v = \frac{2\pi r}{T}$

Solve: $v = \frac{2\pi r}{T} = \frac{2\pi(4.0\text{ m})}{2.0\text{ s}} = 13\text{ m/s}$

b. *Given:* $v = 13$ m/s
$r = 4.0$ m

Unknown: $a_c = ?$
Original equation: $a_c = \frac{v^2}{r}$

Solve: $a_c = \frac{v^2}{r} = \frac{(13\text{ m/s})^2}{4.0\text{ m}} = 42\text{ m/s}^2$

Example 4: Captain Chip, the pilot of a 60 500-kg jet plane, is told that he must remain in a holding pattern over the airport until it is his turn to land. If Captain Chip flies his plane in a circle whose radius is 50.0 km once every 30.0 min, what centripetal force must the air exert against the wings to keep the plane moving in a circle?

Solution: First, convert km to m and min to s.

$$50.0\text{ km} = 5.00 \times 10^4\text{ m} \qquad 30.0\text{ min} = 1.80 \times 10^3\text{ s}$$

Before solving for the centripetal force, find the speed of the airplane.

Given: $T = 1.80 \times 10^3$ s
$r = 5.00 \times 10^4$ m

Unknown: $v = ?$
Original equation: $v = \frac{2\pi r}{T}$

Solve: $v = \frac{2\pi r}{T} = \frac{2\pi(5.00 \times 10^4\text{ m})}{1.80 \times 10^3\text{ s}} = 175\text{ m/s}$

Use this speed to solve for the centripetal force.

Given: $m = 60\,500$ kg
$v = 175$ m/s
$r = 5.00 \times 10^4$ m

Unknown: $F_c = ?$
Original equation: $F_c = \frac{mv^2}{r}$

Solve: $F_c = \frac{mv^2}{r} = \frac{(60\,500\text{ kg})(175\text{ m/s})^2}{5.00 \times 10^4\text{ m}} = 3.71 \times 10^4\text{ N}$

Exercise 8: A cement mixer of radius 2.5 m turns with a frequency of 0.020 Hz. What is the centripetal acceleration of a small piece of dried cement stuck to the inside wall of the mixer?

Answer: ____________

Exercise 9: A popular trick of many physics teachers is to swing a pail of water around in a vertical circle fast enough so that the water doesn't spill out when the pail is upside down. If Mr. Lowell's arm is 0.60 m long, what is the minimum speed with which he can swing the pail so that the water doesn't spill out at the top of the path?

Answer: ____________

Exercise 10: To test their stamina, astronauts are subjected to many rigorous physical tests before they fly in space. One such test involves spinning the astronauts in a device called a *centrifuge* that subjects them to accelerations far greater than gravity. With what linear speed would an astronaut have to spin in order to experience an acceleration of 3 g's at a radius of 10.0 m? (1 g = 10.0 m/s^2)

Answer: ____________

Exercise 11: At the Fermilab particle accelerator in Batavia, Illinois, protons are accelerated by electromagnets around a circular chamber of 1.00-km radius to speeds near the speed of light before colliding with a target to produce enormous amounts of energy. If a proton is traveling at 10% the speed of light, how much centripetal force is exerted by the electromagnets? (Hint: The speed of light is 3.00×10^8 m/s, $m_p = 1.67 \times 10^{-27}$ kg)

Answer: ____________

Exercise 12: Roxanne is making a strawberry milkshake in her blender. A tiny, 0.0050-kg strawberry is rapidly spun around the inside of the container with a speed of 14.0 m/s, held by a centripetal force of 10.0 N. What is the radius of the blender at this location?

Answer: ____________

6-2 Torque

Vocabulary

Torque: A measurement of the tendency of a force to produce a rotation about an axis.

$$\textbf{torque = perpendicular force} \times \textbf{lever arm} \quad \text{or} \quad \tau = F \times d$$

The lever arm, d, is the distance from the pivot point, or fulcrum, to the point where the component of the force perpendicular to the lever arm is being exerted. The longer the lever arm, the larger the torque. This is why it is easier to loosen a tight screw with a long wrench than with your hand or a short pair of tweezers.

If a torque causes a counterclockwise rotation of an object around the fulcrum, it is positive. If the torque causes a clockwise rotation of an object around the